Problem 1:



Figure1: 3 sample paths of the Markov Chain with four states

**Comment:**

The trends of the three paths almost match each other, which show the path visit State 4 most frequently, and then State 3 and State 1, with State 2 less occurred. This result can also be reflected by the transition matrix PI.

**Code:**

clear all;

K=200; % 200 time steps

M=3; % 3 samples

N=4; % 4 states

pi=[0.2 0.2 0.5 0.1;0.2 0.3 0.4 0.1;0.4 0.2 0.3 0.1;0.1 0.0 0.0 0.9];

rand('seed',0);

for j=1:M

i=randsample(1:4,1);

for k=2:K-1

x(k+1,j)= randsample(1:4,1,true,pi(i,:));

i=x(k+1,j);

end

j=j+1;

end

for m=1:M

freq(:,m)=histc(x(:,m),1:N);

end

y=freq./200;

figure (1);

plot(y);

xlabel('four states');

ylabel('probability');

Problem 2:



Figure2: 4 sample paths of the Markov Chain with four states



Figure 3: Relative frequencies with 4 paths visit the four states



Figure 4: Comparison of the estimated relative frequencies with dominant eigenvector

**Comment:**

In Figure 2, the trends of the four paths almost match each other, which show the path visit State 4 most frequently, and then State 2 and State 3, with State 1 less occurred. This result coincides with Figure 3: the relative frequencies with each path visit four states.

In Figure 4, we can see that the plot of relative frequencies almost match the plot of dominant eigenvector. Actually, the dominant eigenvector is the theory/real value for the relative frequencies. This figure shows that the curve of relative frequencies converge to the curve of dominant vector since the transition matrix  is irreducible, a periodic and homogeneous.

**Code:**

clear all;

K=1000; % 1000 time steps

M=4; % 4 samples

N=4; % 4 states

pi=[0.2 0.2 0.1 0.5;0.1 0.3 0.4 0.2;0.3 0.2 0.3 0.2;0.1 0.3 0.1 0.5];

rand('seed',0);

for j=1:M

i=randsample(1:4,1);

for k=2:K-1

x(k+1,j)= randsample(1:4,1,true,pi(i,:));

i=x(k+1,j);

end

j=j+1;

end

% Part a

figure (2);

for m=1:M

freq(:,m)=histc(x(:,m),1:N);

end

y=freq./1000;

plot(y);

xlabel('four states');

ylabel('probability');

% Plot the relative frequencies for four states

figure (3);

for m=1:M

subplot(2,2,m);

hist(x(:,m),1:N);

xlabel('four states');

ylabel('relative frequency');

end

% Part b

% produce eigenvalues (D) and eigenvectors (V) of matrix PI

[V,D]=eig(pi');

ind=find(abs(diag(D)-1)<1e-6);

for k=1:length(ind)

% nv is the rescaled dominant eigenvector

nv(:,k)=V(:,ind(k))/sum(V(:,ind(k)));

end

% Compare the estimated relative frequency with nv

figure(4);

% plot the relative freency

plot(y(:,M),'--');

xlabel('four states');

ylabel('probability');

hold on;

% plot the dominant eigenvector of pi

plot(nv,'--rs');

legend('relative frequency','dominant eigenvector');

xlabel('four states');

ylabel('probability');

Problem 3:

1. ****

Notice: This transition matrix is not irreducible. It has two classes: {state1, state2} and {state 3, state 4}.



Figure 5: 4 sample paths of the Markov Chain with four states



Figure 6: Relative frequencies with 4 paths visit the four states

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Figure 7: Comparison of the estimated relative frequencies with dominant eigenvector

**Comment:**

This Markov Chain is not irreducible, so our theorem that it has a unique probability distribution P such that will not be applied here. Actually, you can find one dominant eigenvector

(Two ‘ind’ values in the program result) for each of the class {state 1, state 2} and {state 3, state 4}. In each of the class, the curve of relative frequencies converges to the curve of dominant vector.

**(b)**

Notice: This transition matrix is not aperiodic. Period=2 for each state.

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Figure 5: 4 sample paths of the Markov Chain with four states

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Figure 6: Relative frequencies with 4 paths visit the four states

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Figure 7: Comparison of the estimated relative frequencies with dominant eigenvector

**Comment:**

This Markov Chain is not aperiodic, so our theorem that it has a unique probability distribution P such that will not be applied here.

In Figure 5, the simulated sample paths are not of the similar trend. But we still can see the four states are equally been visited from Figure 6.

From Figure 7, we can see this relative frequency does not converge to the dominant eigenvector anymore which is due to the insufficient condition for the aperiodic.

**Code:**

clear all;

K=1000;

M=4;

N=4;

%pi=[0.5 0.5 0.0 0.0;0.1 0.9 0.0 0.0;0.0 0.0 0.3 0.7;0.0 0.0 0.2 0.8];

pi=[0.0 0.5 0.0 0.5;0.5 0.0 0.5 0.0;0.0 0.5 0.0 0.5;0.5 0.0 0.5 0.0];

rand('seed',0);

for j=1:M

i=j;

% randsample(1:4,1);

for k=2:K-1

x(k+1,j)= randsample(1:4,1,true,pi(i,:));

i=x(k+1,j);

end

j=j+1;

end

figure (5);

for m=1:M

freq(:,m)=histc(x(:,m),1:N);

end

y=freq./1000;

plot(y,'LineWidth',2);

xlabel('four states');

ylabel('probability');

figure (6);

for m=1:M

subplot(2,2,m);

hist(x(:,m),1:N);

xlabel('four states');

ylabel('relative frequency');

end

% produce eigenvalues (D) and eigenvectors (V) of matrix pi

[V,D]=eig(pi');

ind=find(abs(diag(D)-1)<1e-6);

for k=1:length(ind)

% nv is the rescaled dominant eigenvector

nv(:,k)=V(:,ind(k))/sum(V(:,ind(k)));

end

% plot the relative frequency

figure (7);

plot(y(:,M),'--','LineWidth',3);

xlabel('four states');

ylabel('probability');

hold on;

% plot the dominant eigenvector of pi

plot(nv,'--rs');

legend('relative frequency','dominant eigenvector');

xlabel('four states');

ylabel('probability');